A new expression for the form stress term in the vertically Lagrangian mean framework for the effect of surface waves on the upper ocean circulation

H. Aiki[1] and R. J. Greatbatch[2]

1 Japan Agency for Marine-Earth Science and Technology, Japan

2 GEOMAR, Germany

(aiki@jamstec.go.jp, http://www.jamstec.go.jp/frcgc/research/d1/aiki/)

Abstract

There is an ongoing discussion in the community concerning the wave-averaged momentum equations in the hybrid vertically Lagrangian and horizontally Eulerian (VL) framework (Mellor, 2003; Aiki and Greatbatch, 2012) and, in particular, the form stress term (representing the residual effect of pressure perturbations) which is thought to restrict the handling of higher order waves in terms of a perturbation expansion (Ardhuin et al. 2008; Aiki and Greatbatch, 2013). The present study shows that the traditional pressure-based form stress term can be transformed into a set of terms that do not contain any pressure quantities but do contain the time derivative of a wave-induced velocity. This wave-induced velocity is referred to as the pseudomomentum in the VL framework, as it is analogous to the generalized pseudomomentum in Andrews and McIntyre (1978). This enables the second expression for the wave-averaged momentum equations in the VL framework (this time for the development of the total transport velocity minus the VL pseudomomentum) to be derived together with the vortex force. The velocity-based expression of the form stress term also contains the residual effect of the turbulent viscosity, which is useful for understanding the dissipation of wave energy leading to transfer of momentum from waves to circulation. It is found that the concept of the virtual wave stress of Longuet-Higgins (1953) is applicable to quite general situations: it does not matter whether there is wind forcing or not, the waves can have slow variations, and the viscosity coefficient can vary in the vertical. These results provide a basis for revisiting the surface boundary condition used in numerical circulation models.

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