**A wave action equation for water waves propagating on vertically sheared flows**

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Coexistence of motions of different scales in oceans and other natural water basins presents a challenge for their dynamic modeling. For water waves-on-currents context the most common approach exploits the disparity of scales to separate equations describing propagation of ``fast/short" surface waves in the slowly varying environment and equations for ``slow/long" currents. This approach can be formulated mathematically as an asymptotic procedure exploiting the existing separation of scales. In the leading order it allows one to tackle separately two motions of a qualitatively different nature. To describe wave propagation over distances far exceeding the characteristic wavelength, the corresponding wave evolution equations are further simplified by phase averaging. This yields an evolution equation for wave action, the latter depends only on slow space and time variables. The commonly used wave action equation is restricted to slowly varying bottom depths and vertically averaged ambient currents.

In recent years the capability of circulation models has significantly improved. In particular, in contrast to the earlier vertically averaged representation of currents, the modern models reproduce the vertical variability of the flows. Further advancements have coupled wave action models to circulation ones in order to account for the mutual effects between the two types of motion resulting in a more wholesome approach. Nevertheless, the commonly used wave action equation neglects important aspects of real oceanic flows. Water waves in oceans and other natural basins almost always propagate on currents with a pronounced vertical structure; this structure might differ significantly from the depth-average approximation. The primary goal of this work is to derive and examine a general wave action equation that accounts for the above shortcoming. The developed wave action formulation greatly improves the representation of linear wave-current interaction in the case of tidal inlets, wind-induced currents, storm surges and undertow currents.

In contrast to the case of vertically averaged ambient currents, the structure of the oscillatory flow under the wave depends on the current's vertical structure. Locally, this structure is described by a solution of the Rayleigh equation with appropriate surface and bottom boundary conditions---an essential step for creating an applicable explicit wave action formulation. As for arbitrary current profile the Rayleigh equation boundary-value problem does not have an exact analytical solution, two asymptotic

solutions are employed and discussed. The first is a perturbation solution of Rayleigh's equation for an arbitrary water depth, which assumes small curvatures and gradients of ambient current but without a limitation on the current's magnitude. The second does not impose limitations on the magnitude of current vertical shear. The advantages of the derived wave action equation using these two vertical asymptotic solutions are discussed by analyzing the numerical solution of simple examples of wave-current interaction problem.