

Wave-current interaction in strongly sheared mean flows

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Abstract

We describe a framework for wave-current interaction theory for small-amplitude surface gravity waves propagating on the strongly sheared mean flows. Using a multiple-scale perturbation method, we derive a couple system of equations for waves and mean flow. In the presence of an arbitrarily-sheared mean flow, wave motion is governed at leading order by the Rayleigh (or inviscid Orr-Sommerfeld) equation. A wave action equation follows from consideration of a solvability condition at second order. In turn, wave forcing effects on the mean flow are formulated using a vortex-force formulation, extending previous work of McWilliams et al (2004) to the case of arbitrary current shear.

We further consider a framework for applying the theory in practice. The Rayleigh stability equation is solved using either a numerical method or perturbation method. Results illustrate that the vertical structure of mean flow modifies the wave orbital velocity and generates wave vorticity. A comparison of both numerical solution and perturbation approximation based on measured current velocity profiles at the mouth of Columbia river indicates that the second order perturbation solution successfully captures the main features of current shear effects on wave vertical structure. Using a representation of the mean current field in terms of fitted polynomials over depth, the perturbation solution for the vertical wave structure is then used to obtain expressions for wave action density, action flux, and wave forcing terms which are then functions of horizontal position (plus vertical in wave forcing terms) and the polynomial coefficients of the current field.